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Abstracts of Papers to Appear in Future Issues

HYDRODYNAMIC MODELING OF PARTICLE AND ANGULAR MOMENTUM TRANSPORT IN ROTATING TOKAMAK PLASMAS WITH IMPURITIES. R. Zanino, *Energetics Department, Polytechnic Institute of Turin, ITALY.*

We have developed a 1 + 1 D time dependent code for the description of ion-impurity transport in a rotating tokamak plasma, using a pseudo-spectral discretization in the poloidal angle θ and a staggered finite difference mesh in the minor radius r. The plasma is assumed to have a constant uniform temperature T, to be in the high collisionality (Pfirsch–Schlüter) regime, and to contain electrons "e," fuel ions "i," and a single impurity species "Z" of charge eZ, where e is the proton charge. We are particularly interested in the case when: (1) flow velocities in the toroidal (symmetry) direction ϕ are in the range typical of neutral beam injection experiments, i.e.,

$$v_{thZ} < V_{\phi i,Z} \lesssim v_{thi}$$

($v_{thj} \equiv \sqrt{2T/m_j}$ is the thermal speed, m_j is the mass); (2) the relative concentration of impurities in the plasma, $\bar{n}_Z/\bar{n}_i$, is significant and comparable to that observed in present tokamaks, i.e.,

$$\sqrt{\frac{m_e}{m_i}} \ll \frac{\bar{n}_Z Z^2}{\bar{n}_i} \approx 1$$

in order of magnitude. The model fluid equations are obtained via a moment approach and an expansion in powers of the small ordering parameter

$$\delta_{pi} \equiv \left(\frac{m_i v_{thi}}{eB_0}\right)\left(\frac{1}{\bar{n}_i}\left|\frac{\partial \bar{n}_i}{\partial r}\right|\right) \ll 1$$

(B is the magnetic field) is then employed. The equations at each order in δ_{pi} up to second order are solved, and the characteristic features of the results are presented: to lowest order, outboard impurity peaking on each magnetic surface appears due to centrifugal forces; to first order, radial gradients driven ion-impurity friction gives rise to up-down asymmetries in the poloidal profiles; to second order, the radial profiles of density and rotation frequency evolve to steady state under the action of particle and angular momentum sources. The evolution of the poloidal profiles is decoupled from the evolution of the radial ones, thanks to the fact that the corresponding time scales belong to different orders in δ_{pi}: an algorithm is proposed to treat the 2D problem, alternating the solution of 1D problems.

AN ANTI-SPATIAL ALIASING FILTER FOR EXPLICIT MODELING AND IMAGING IN INHOMOGENEOUS MEDIA. Alvin K. Benson, *Department of Geology and Geophysics, Brigham Young University, Provo, Utah 84602, USA.*

An explicit finite difference solution to the scalar wave equation in isotropic, inhomogeneous media is completed by filtering out nonphysical contributions to the data. This digital, anti-spatial-aliasing filter and some associated limits on angular frequency are determined. The filter is a projection operator determined from a constained least-squares fit and can be implemented in the computer algorithm at either

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of two places. Furthermore, the filter should be applicable to *any* explicit finite difference solution to the wave equation. Unlike a standard dip filter, this filter is computationally flexible, efficient, and necessary in inhomogeneous media with rapid lateral and vertical velocity changes.

STABILITY ANALYSIS OF A NON-LINEAR DIFFUSION-TYPE KINETIC EQUATION. Henry Granek, *School of Physics, University of Melbourne, Parkville, Victoria, AUSTRALIA and CSIRO, Division of Atmospheric Research, Aspendale, Victoria, AUSTRALIA*; Bruce H. J. McKellar, *School of Physics, University of Melbourne, Parkville, Victoria, AUSTRALIA.*

A diffusion-type partial differential equation with non-linear coefficients is analysed for stability in the von Neumann sense, and some numerical examples given. The equation is a kinetic equation representing an instantaneous injection of energetic photons into a thermalised cosmological background radiation (CBR) and the subsequent time evolution of the electromagnetic spectrum. Compton, Double Compton and Bremsstrahlung are the only interactions considered at the relatively photon low energies. The final conservative, implicit, finite difference scheme is a refinement of a similar model developed by Lightman, which is shown to be not stable for some cases considered. A semi-Lagrangian modification is used to account for the expansion of the universe. The full physical derivation of the kinetic equation and the associated parameters are given elsewhere.

RAPID DETERMINATION OF A STRAIGHT MAGNETIC COORDINATE SYSTEM FOR STELLARATOR CONFIGURATIONS. D. K. Lee and S. P. Hirshman, *Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA.*

A flux coordinate representation for the magnetic field is used to derive a system of simultaneous linear equations for accurately computing the rotational transform and the poloidal angle stream function for a given magnetic flux surface in a toroidal stellarator configuration. The procedure is useful for converting an arbitrary flux coordinate system into one for which all magnetic field lines are straight. It is quite general and requires only that the flux surfaces can be represented by double Fourier series of the coordinates R and Z and that values of the cylindrical components of the magnetic field are available on each surface. Numerical results obtained for vacuum configurations of the Advanced Toroidal Facility (ATF) show that the present procedure is more accurate and convenient than previous methods.

NUMERICAL SOLUTION OF THE STEADY STOKES EQUATIONS. Eric Yu Tau, *Lawrence Berkeley Laboratory, Department of Mathematics, University of California, Berkeley, California 94720, USA.*

In this paper we present a fast numerical technique for finding solutions of the steady-state Stokes equations on both two- and three-dimensional domains. We implement the method on a special staggered grid for a rectangular (cubic) domain and obtain a solution in an order of $O(N \log N)$ operations for both two- and three-dimensional cases, where N is the number of grid points in the domain. The main idea is to derive from the Stokes equations an equation for the pressure p, $Ap = b$, where the matrix A is semipositive definite and very well conditioned on the orthogonal complement of its null space.

SIXTH-ORDER LIE GROUP INTEGRATORS. Etienne Forest, *Exploratory Studies Group, Lawrence Berkeley Laboratory, 1 Cyclotron Road, Berkeley, California 94720, USA.*

In this paper we present the coefficients of several sixth-order symplectic integrator of the type developed by R. Ruth. To get these results we fully exploit the connection with Lie groups. This integrator, as well as all the explicit integrators of Ruth, may be used in any equation where some sort of Lie bracket is preserved. In fact, if the Lie operator governing the equation of motion is separable into two solvable parts, the Ruth integrators can be used.

LOCALIZATION SCHEMES IN 2D BOUNDARY-FITTED GRIDS. Thomas Westermann, *Kernforschungszentrum Karlsruhe GmbH, Abteilung fur Numerische Physik, HDI-3, P.O. Box 3640, 7500 Karlsruhe, GERMANY.*

A discussion of localization schemes in two-dimensional structured grids consisting of convex four-point meshes is presented. These algorithms are applicable to particle-in-cell codes based on two-dimensional boundary-fitted coordinates in order to localize particles inside the grid. They are fully vectorizable and two of them are directly applicable also to triangular meshes. Since all of them are exact, they avoid an overhead for a special treatment of particles near the boundary as it is necessary for the approximate localization proposed by Seldner and Westermann (*J. Comput. Phys.* **79**, 1–11 (1988)). Hence, they are suitable for complicated geometries with outer and inner curved boundaries. Depending on the vector computer used, a speedup of 3.5 to 8 is achieved for the fastest algorithm.

TIME DOMAIN NUMERICAL CALCULATIONS OF UNSTEADY VORTICAL FLOWS ABOUT A FLAT PLATE AIRFOIL. S. I. Hariharan and Yu Ping, *Department of Mathematical Sciences, University of Akron, Akron, Ohio 44325, USA*; J. R. Scott, *NASA Lewis Research Center, Cleveland, Ohio 44135, USA.*

A time domain numerical scheme is developed to solve for the unsteady flow about a flat plate airfoil due to imposed upstream, small amplitude, transverse velocity perturbations. The governing equation for the resulting unsteady potential is a homogeneous, constant coefficient, convective wave equation. Accurate solution of the problem requires the development of approximate boundary conditions which correctly model the physics of the unsteady flow in the far field. An accurate far field boundary condition is developed, and numerical results are presented using this condition. The stability of the scheme is discussed, and the stability restriction for the scheme is established as a function of the Mach number. Finally, comparisons are made with the frequency domain calculations by Scott and Atassi, and the relative strengths and weaknesses of each approach are assessed.

THE INCLUSION OF COLLISIONAL EFFECTS IN THE SPLITTING SCHEME. L. Demeio, *Center for Transport Theory and Mathematical Physics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061, USA.*

An algorithm is given for the numerical solution of the Boltzmann equation for a one-dimensional unmagnetized plasma with immobile ions, in which collisional effects are described by the Bhatnagar–Gross–Krook (BGK) model. The algorithm is a straightforward generalization of the splitting scheme, which solves the one-dimensional Vlasov–Poisson system. The accuracy of the splitting scheme to second order in Δt is preserved.

BOX-COUNTING ALGORITHM AND DIMENSIONAL ANALYSIS OF A PULSAR. F. H. Ling and G. Schmidt, *Department of Physics and Engineering Physics, Stevens Institute of Technology, Hoboken, New Jersey 07030, USA.*

It is argued that the use of the box-counting algorithm to calculate the correlation dimension is a better choice than the Grassberger–Procaccia (correlation integral) algorithm for dealing with an experimental data set. This is illuminated by treating three classical examples: the logistic map, the Hénon map, and the Lorenz equation. The intensity data of a pulsar is also treated which is revealed to have a least embedding dimension of 14 and the correlation dimension of about 4.5.

NOTES TO APPEAR

SOLUTION OF THE SHALLOW WATER EQUATIONS USING HYBRID GRIDS. J. Steppeler, *National Meteorological Center, Washington, DC 20233, USA.*

TWO-POINT QUASI-FRACTIONAL APPROXIMATIONS TO THE AIRY FUNCTION $Ai(x)$. Pablo Martin, *Depto. de Fisica, Universidad Simon Bolivar, Apartado 89000, Caracas 1086, VENEZUELA*; Ricardo Perez, *Depto. de Estudios Preparatorios, Instituto Universitario de Tecnologia "Dr. Federico Rivero Palacio," Apartado 40347, Caracas 1040, VENEZUELA*; Antonio L. Guerrero, *Depto. de Fisica, Universidad Simon Bolivar, Apartado 89000, Caracas 1086, VENEZUELA.*

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